30

THE CLAIMS DEFINING THE INVENTION ARE AS FOLLOWS:

1.   A method for estimating the frequency of a single frequency complex exponential tone in additive Gaussian noise, comprising the steps of:

5   performing the fast Fourier transform (FFT) on the tone;

estimating the frequency as the frequency corresponding to the largest FFT output coefficient magnitude;

computing a discriminant which is proportional to the frequency error in the initial frequency estimate using modified coefficients of the discrete Fourier

10   transform (DFT) with center frequencies plus one half and minus one half of the FFT bin spacing relative to the initial frequency estimate;

mapping the value of the discriminant into the estimate of the frequency error in the initial frequency estimate using a mathematically derived function;

adding the estimate of the frequency error to the initial frequency

15   estimate to get a first interpolated frequency estimate;

computing a further discriminant which is proportional to the frequency error in the first interpolated frequency estimate using modified coefficients of the discrete Fourier transform (DFT) with center frequencies plus one half and minus one half of the FFT bin spacing relative to the first interpolated frequency

20   estimate;

mapping the value of the further discriminant into the estimate of the frequency error in the first interpolated frequency estimate using the mathematically derived function; and

adding the estimate of the frequency error in the first interpolated

25   frequency estimate to the first interpolated frequency estimate to get a second interpolated frequency estimate.

2.   The method according to claim 1, wherein the first interpolated frequency estimate is in a region of relatively low noise induced frequency error.

3.   The method according to claim 1 or 2, wherein the method is

30   implemented in computer hardware and/or computer software.

4.   The method according to any one of the preceding claims, wherein the method is utilised in communications, signal processing and biomedical applications.

5.   The method according to any one of the preceding claims, further

35   comprising the steps of:

iteratively deriving an interpolated frequency estimate, and

using the frequency discriminant, to obtain a more precise frequency estimate.

6.   The method according to claim 5, wherein the steps of iteratively deriving an interpolated frequency estimate and using the frequency discriminant are repeated until a fixed point solution occurs, where at this fixed point, the discriminant function has zero value.

7.   The method according to any one of the preceding claims, wherein the frequency discriminant is computed by:

$$D(\varepsilon, \hat{\varepsilon}) = \frac{|\beta| - |\alpha|}{|\beta| + |\alpha|}$$

where, $\varepsilon = fT_s - \dfrac{k_{max}}{N}$,

$\hat{\varepsilon} = \hat{f}T_s - \dfrac{k_{max}}{N}$, and

β and α are the modified DFT coefficients defined by,

$$\beta = Y(k_{max} + \frac{1}{2}) = \sum_{n=0}^{N-1} r(n)e^{-j2\pi n\frac{(k_{max}+\frac{1}{2})}{N}}$$

and, $\alpha = Y(k_{max} - \frac{1}{2}) = \sum_{n=0}^{N-1} r(n)e^{-j2\pi n\frac{(k_{max}-\frac{1}{2})}{N}}$

thus, the initial frequency estimate using the FFT, $\hat{f}_0 T_s = \dfrac{k_{max}}{N}$ and $\hat{\varepsilon} = 0$.

8.   The method according to any one of claims 1 to 6, wherein the frequency discriminant is computed by:

$$D = \frac{1}{\gamma} \frac{|\beta|^\gamma - |\alpha|^\gamma}{|\beta|^\gamma + |\alpha|^\gamma}, \quad \text{for } \gamma > 0.,$$

where β and α are the modified DFT coefficients defined by,

$$\beta = Y(k_{max} + \frac{1}{2}) = \sum_{n=0}^{N-1} r(n)e^{-j2\pi n\frac{(k_{max}+\frac{1}{2})}{N}}$$

and, $\alpha = Y(k_{max} - \frac{1}{2}) = \sum_{n=0}^{N-1} r(n)e^{-j2\pi n\frac{(k_{max}-\frac{1}{2})}{N}}$

9.   The method according to any one of claims 1 to 6, wherein the discriminant of frequency estimation error is computed by:

$$D = \frac{1}{2} \frac{|\beta|^2 - |\alpha|^2}{|\beta|^2 + |\alpha|^2},$$

32

where β and α are the modified DFT coefficients defined by,

$$\beta = Y(k_{max} + \frac{1}{2}) = \sum_{n=0}^{N-1} r(n)e^{-j2\pi n \frac{(k_{max} + \frac{1}{2})}{N}}$$

and, $\alpha = Y(k_{max} - \frac{1}{2}) = \sum_{n=0}^{N-1} r(n)e^{-j2\pi n \frac{(k_{max} - \frac{1}{2})}{N}}$

10.   The method according to any one of claims 1 to 6, wherein the frequency discriminant is computed by:

$$D = Re[\frac{\beta - \alpha^*}{\beta + \alpha^*}]$$

where Re[.] is the real part and * denotes the complex conjugate, and β and α are the modified DFT coefficients defined by,

$$\beta = Y(k_{max} + \frac{1}{2}) = \sum_{n=0}^{N-1} r(n)e^{-j2\pi n \frac{(k_{max} + \frac{1}{2})}{N}}$$

and, $\alpha = Y(k_{max} - \frac{1}{2}) = \sum_{n=0}^{N-1} r(n)e^{-j2\pi n \frac{(k_{max} - \frac{1}{2})}{N}}$

11.   The method according to any one of preceding claims, wherein the frequency discriminant is computed by using more than two DFT coefficients.

12.   The method according to claim 11, wherein 2M+2 coefficients are used, where $0 \le M \le \frac{N}{2} - 1$ and the FFT coefficients are used in the frequency discriminant with optimal weighting coefficients obtained by using the concept of matched filtering is,

$$D = Re[\frac{\sum_{m=0}^{M} C_{[k_{max} + \frac{1}{2} + m]}\{Y[(k_{max} + \frac{1}{2} + m) \bmod N] - Y^*[(k_{max} - \frac{1}{2} - m) \bmod N]\}}{\sum_{m=0}^{M}\{C_{[k_{max} + \frac{1}{2} + m]}\{Y[(k_{max} + \frac{1}{2} + m) \bmod N] + Y^*[(k_{max} - \frac{1}{2} - m) \bmod N]\}}]$$

where, $0 \le M \le \frac{N}{2} - 1$, mod N indicates modulo N,

and, where, * denotes complex conjugate.

$$C_{k_{max} + \frac{1}{2} + m} = \frac{e^{j\pi[\frac{1}{2} + m][\frac{N-1}{N}]} \sin[\pi(\frac{1}{2} + m)]}{\sin[\frac{\pi}{N}(\frac{1}{2} + m)]}$$

and, $Y(k_{max} + \frac{1}{2} + m)$ and $Y(k_{max} - \frac{1}{2} - m)$ are the modified DFT coefficients given by,

$$Y(k_{max} + \frac{1}{2} + m) = \sum_{n=0}^{N-1} r(n) e^{-j2\pi n \frac{(k_{max} + \frac{1}{2} + m)}{N}}$$

and, $$Y(k_{max} - \frac{1}{2} - m) = \sum_{n=0}^{N-1} r(n) e^{-j2\pi n \frac{(k_{max} - \frac{1}{2} - m)}{N}}$$

13.　The method according to claim 11 or 12, wherein the frequency discriminant using more than two DFT coefficients us used in the last iteration to obtain additional frequency accuracy.

14.　The method according to any one of claims 11 to 13, wherein the frequency discriminant is computed by using more than two DFT coefficients and less or equal to all N FFT coefficients.

15.　The method according to any one of the preceding claims, wherein additional frequency accuracy is obtained by computing the frequency discriminant recursively until convergence for the frequency estimate is reached.

16.　The method according to claim 15, wherein convergence for the frequency estimate is reached after zero to three iterations, the number of iterations being dependent on the specific discriminant used and the signal to noise ratio.

17.　The method according to claim 15 or 16, wherein in any iteration, the frequency discriminant is computed using any one of the functional forms:

$$\Delta f_m(r) = \frac{1}{\pi} \tan^{-1}[\frac{|\beta_m| - |\alpha_m|}{|\beta_m| + |\alpha_m|} \tan(\frac{\pi}{2N})] f_s, \text{ or}$$

$$\Delta f_m(r) = \frac{1}{2N\gamma_m}[\frac{|\beta_m|^{\gamma_m} - |\alpha_m|^{\gamma_m}}{|\beta_m|^{\gamma_m} + |\alpha_m|^{\gamma_m}}] f_s, \text{ where } \gamma_m \text{ is a constant}, \gamma_m > 0, \text{ or}$$

$$\Delta f_m(r) = \frac{1}{2N}[\frac{|\beta_m| - |\alpha_m|}{|\beta_m| + |\alpha_m|}] f_s, \quad \text{for } \gamma = 1, \text{ or}$$

$$\Delta f_m(r) = \frac{1}{4N}[\frac{|\beta_m|^2 - |\alpha_m|^2}{|\beta_m|^2 + |\alpha_m|^2}] f_s, \quad \text{for } \gamma = 2.$$

18.　The method according to claim 17, wherein γ varies on each iteration.

19.　The method according to claim 15 or 16, wherein in any iteration, the frequency discriminant is computed using:

$$\Delta f_m(r) = \frac{1}{2N} Re[\frac{\beta_m - \alpha_m^*}{\beta_m + \alpha_m^*}] f_s,$$

where, Re[.] denotes the real part and * denotes the complex conjugate.

20.   The method according to any one of claims 17 to 19, wherein the frequency incremental shift, $\Delta f_m(r)$, is related to the previously defined frequency discriminant, D, by,

$$\Delta f_m(r) = \frac{f_s}{2N} D$$

21.   The method according to any one of claims 15 to 20, wherein the frequency discriminant is driven to zero input and output values by either modifying the frequency of the DFT coefficients or frequency translating the signal.

22.   The method according to any one of claims 15 to 21, wherein signal frequency translation is achieved by multiplication of the signal by a locally generated complex exponential signal.

23.   The method according to claim 22, wherein frequency multiplication of the signal is implemented with a standard hardware, software, or combination hardware/software FFT.

24.   The method according to claim 23, wherein the hardware/software FFT is highly optimized for at least one processor operating as a system.

25.   The method according to any one of claims 15 to 24, further comprising the step of scaling the frequency estimate during recursion, to save multiplies.

26.   The method according to claim 25, further comprising a final step of multiplying the scaled frequency estimate $\hat{f}_{m+1}T_s$ with the sampling frequency f_s to remove the scaling from the frequency estimate.

27.   A frequency estimation software program for estimating the frequency of a single frequency complex exponential tone in additive Gaussian noise, wherein the frequency estimation program has functionality to perform the method according to any one of claims 1 to 26.

28.   A computer system programmed to perform the method according to any one of claims 1 to 26.

29.   The computer system according to claim 28, wherein the hardware includes a DSP processor chip.